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Back action of a low noise dc SQUID

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Measurements are presented of the back action of a low noise commercial dc superconducting quantum interference device (SQUID) on a strongly-coupled high-quality factor ($Q \approx 10^6$) electrical LC resonator operating at audio frequencies ($\approx$1 kHz). The back-action effect, due to the voltage noise of the SQUID current amplifier, is in good agreement with the predictions. The value of the noise impedance of the SQUID coincides, within 5%, with its input coil inductance times the angular frequency. © *1998 American Institute of Physics.* [S0003-6951(98)02750-8]

The coupled dc superconducting quantum interference device (SQUID) [see Fig. 1(a)] is usually treated as a current amplifier, whose input impedance is a small inductance $L_i \approx 1$ μH and whose noise is given by an input current noise source I_n. A widely used figure of merit for the SQUID in this configuration is its energy resolution per unit bandwidth referred to the input coil, $\epsilon = S_i L_i$ where S_i is the double-sided spectral density of the noise source I_n. In literature there are reports of SQUIDs coupled to an input coil with energy resolution near the quantum limit $\hbar$,[1] demonstrating that this device is the most sensitive low-frequency amplifier. For that reason, SQUIDs are employed as first-stage amplifiers in several fundamental physics experiments, either planned or in progress, including space-based gravitational experiments,[2] cryogenic resonant gravitational wave detectors,[3] and weakly interacting massive particle detectors.[4]

In the experiments in which the signal source impedance in the input circuit is resonant with a high quality factor, this simple model for the dc SQUID is not sufficient, with respect to input impedance and noise sources, and a more general model is needed. The dynamic input impedance can be modeled as an inductance and a resistance in series with L_i, both depending on the flux applied to the SQUID.[5] In closed-loop operation, the impedance must be known, and possibly controlled by means of a suitable feedback network in order to avoid instabilities of the resonator–SQUID system. The effect of the input circuit on the SQUID output noise can be treated by associating two partially correlated noise sources with an ideal noise-free current amplifier: a current noise source in parallel with the input port and a voltage noise source in series with it [Fig. 1(b)]. The current noise is superimposed on the input current signal and the voltage noise produces a driving force (back action) on the signal source. Because a correctly applied negative feedback does not change the signal-to-noise ratio, this model remains valid for closed-loop operation. This class of experiments includes, for example, the resonant gravitational wave detectors and the NMR tuned radiofrequency amplifier[6] for which, in order to obtain the noise matching between the SQUID and the detector, knowledge of the characteristics of both noise sources is of fundamental importance.

The current noise measurement is easily realized by measuring the SQUID output voltage noise and by knowing the values of the mutual inductance M_i between the input coil and SQUID loop and the SQUID flux-to-voltage gain. The expected value[7] of the voltage noise spectral density S_v is related to the current noise spectral density S_i by $S_v \cong (\omega L_i)^2 S_i$. To our knowledge, all measurements of S_v have been performed on a dc SQUID fabricated with shunt resistances much less than optimum, so such noise is enhanced by two orders of magnitude.[8] In this letter we report the voltage noise measurement of a low noise commercial dc SQUID[9] realized by strongly coupling the SQUID to an electrical resonator with a variable quality factor [Fig. 1(a)].

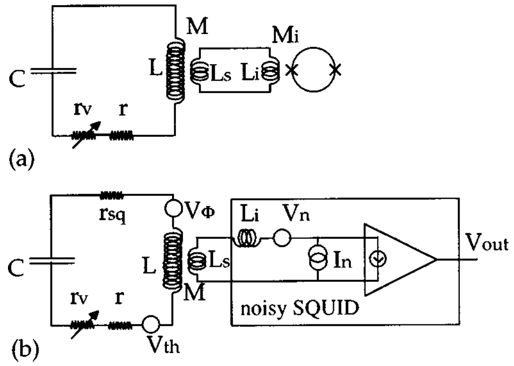

FIG. 1. (a) The schematic circuit diagram of the SQUID coupled to the resonator with variable quality factor and (b) its circuit model; the dc SQUID is represented by a noisy current amplifier and the real part of its dynamic input impedance by the ''cold'' resistance r_{sq}. The manufacturer SQUID specifications are $L_i = 1.88$ μH and $M_i = 11$ nH; $M = 0.56$ mH and $L_s \cong 1.8$ μH.

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In Fig. 1(b), in addition to the current and voltage noise sources I_n and V_n, which are assumed to be uncorrelated,[10] we include a voltage thermal noise source $V_{\rm th}$, with spectral density $2k_BT(r+r_v)$, associated with resonator losses. The voltage noise source V_Φ (spectral density S_Φ), accounting for any magnetic field flux noise picked up by the coil, is the more probable spurious noise source; this may be due to the ambient flux noise which reaches the coil or to the vibrations of the coil with respect to the magnetic field trapped within its superconducting housing. The input impedance of the dc SQUID is represented by a pure inductance L_i; the effect of the real part of the dynamic input impedance is represented by a positive or negative resistance $r_{\rm sq}$, which adds to the resistances r and r_v. In the model $r_{\rm sq}$ affects the resonator quality factor without adding any noise, that is, it realizes so-called cold damping. For the purpose of this measurement, the imaginary part of the impedance has a negligible effect. The noise power spectrum of the SQUID output can be roughly described by the sum of a broadband noise, due to S_i, and a (Lorentzian shape) peak noise which rises above the broadband noise by up to ten orders of magnitude. Following the model, it is possible to compute the variance of the peak noise of the SQUID output, which is the sum of three contributions: the first is due to the thermal noise of the resonator losses, the second to the flux noise picked up by the coil, and the third to the SQUID voltage noise:

$$\sigma^2_{\rm out}=\frac{Q_a}{Q_i}(MT_RG)^2 \times\left\{\frac{k_BT}{L_r}+Q_i\left[\frac{S_\phi}{2\omega_0L_r^2}+\frac{S_V}{2\omega_0}\left(\frac{M}{L_tL_r}\right)^2\right]\right\}, \qquad (1)$$

where $L_t=L_i+L_s$, the transformer ratio $T_R=M_i/L_t$, the inductance of the coil reduced by the coupling to the SQUID $L_r=L-M^2/L_t$, the resonance angular frequency $\omega_0=2\pi\nu_0\cong(L_rC)^{-1/2}$, the intrinsic quality factor $Q_i=\omega_0L_r/(r+r_v)$, the measured quality factor, which is determined also by the SQUID dynamic input impedance, $Q_a=\omega_0L_r/(r+r_v+r_{\rm sq})$, and G is the flux-to-voltage gain of the SQUID. The factor $(MT_RG)^2$ transforms the coil current noise to the output voltage noise of the SQUID and can be estimated with a calibration measurement. Given the resonance frequency and coupling, it is possible to distinguish between the thermal noise and the sum of the other two noise contributions by varying the intrinsic quality factor of the resonator and measuring the peak noise, as described in detail below.

The resonator is based on a low-loss, low-stray-capacitance superconducting coil[11] with inductance (in its superconducting housing) $L=0.554\pm0.005$ H. In order to change the operation frequency, two different Teflon capacitors (51.9 and 25.4 nF) were used. The resonator and the SQUID are housed in separated superconducting shielding cases and the resonator Q_i can be reduced, from a maximum value of 1.2 million to 20 000, by progressively inserting in the coil, by means of a room-temperature motion feedthrough, a small copper cylinder 15 mm in length and 2 mm in diameter (the dissipator), located in a PVC rod.[12]

The measurement method permits, by means of a suitable filtering, to make negligible the I_n broadband contribution to the SQUID output noise variance and, then, to use Eq. (1) for the noise analysis. The SQUID output is sent to a lock-in amplifier tuned on the resonator resonance frequency so that it operates as a bandpass filter. The lock-in time constant (0.1–0.3 s) is adjusted to give an equivalent bandwidth wide enough to contain the resonator noise bandwidth but narrow enough to make negligible the broadband noise contribution. The two lock-in phases X_n and Y_n are sampled every two resonator decay times in order to have uncorrelated data sets and, from the exponential weighted fit of the histogram of $R_n^2=X_n^2+Y_n^2$, the peak noise variance and its error are estimated. The variance measurement is repeated at different Q_i, that is, at different dissipator positions.

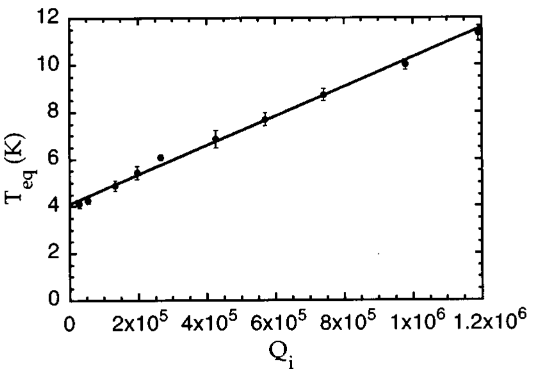

FIG. 2. The peak noise variance at 1476 Hz, expressed as equivalent temperature $T_{\rm eq}$ vs the resonator intrinsic quality factor Q_i.

To derive S_v from the peak noise variance it is necessary to know Q_a, ω_0, Q_i, MT_RG, and L_r. The measurement of Q_a and ω_0 is carried out by analyzing the magnitude and phase data measured by a lock-in tuned, approximately, on ω_0 during a resonator free decay. The intrinsic quality factor Q_i is evaluated from the calibration curve Q_i versus the dissipator position realized, by measuring the resonator quality factors with a SQUID so weakly coupled that it negligibly affects the intrinsic quality factor. For a detailed description of this measurement method, see Ref. 13. To measure MT_RG, a calibration run is performed, in which a small known current is injected in the coil (disconnected from the capacitors) and the SQUID response is measured. During this same run we measured also the low-temperature value of the capacitance; from this and the resonance frequency the inductance L_r of the coil coupled to the SQUID has been evaluated.

All the noise measurements at different Q_i and the measurements for the determination of the curve Q_i versus the dissipator position have been performed for the two resonator resonance frequencies, 1032 and 1476 Hz.

Figure 2 shows the peak noise variance at 1476 Hz, expressed as equivalent temperature $T_{\rm eq}=\sigma^2_{\rm out}(Q_i/Q_a)\times(1/MT_RG)^2(L_r/k_B)$, as a function of the intrinsic quality factor. As expected from the model [Eq. (1)], the data are well described by a straight line whose zero Q_i intercept (4.1 ± 0.1 K) is compatible, within the error, with the temperature (4.19 K) measured by a calibrated thermometer located near the SQUID sensor. The same equivalent temperature value, although less precise (4.2 ± 0.2 K), is found with a linear fit of the data taken at 1032 Hz. This means that the ''cold damping'' hypothesis is verified, that all the dissipative elements of the resonator which contribute to determining the Q_i are well thermalized and that the calibration measurements, carried out to estimate MT_RG and L_r, are accurate. From the slopes of the linear fits of the equivalent temperatures as functions of Q_i, it is possible to evaluate the values of S_v at the two operation frequencies only by assuming that all noise in excess of thermal noise is due to the SQUID back action. This hypothesis is supported by measurements[12] performed in the same frequency range with the same apparatus, but with a weaker coupling between resonator and SQUID. These measurements showed that the slope of the equivalent temperature as a function of Q_i is zero within 0.2 K. As the ratio of the thermal and flux noise contribution is, to a good approximation, independent of the coupling, it can be concluded that the superconducting shield of the resonator and the system vibration rejection are sufficient to keep S_Φ at a negligible level.

The weighted mean of the two values of $S_v^{1/2}/\omega_0 S_i^{1/2}$ is $1.8\pm0.1\ \mu\text{H}$, which is in good agreement with the expected value of $1.88\ \mu\text{H}=L_i$. Because the dc SQUID noise theory concerns only the sensor and not the following stage amplifiers, the value of S_i which has been employed in the calculation of $S_v^{1/2}/\omega_0 S_i^{1/2}$ corresponds to the SQUID output broadband noise from which the preamplifier noise has been subtracted. This value, $S_i=(1.2\pm0.1)\times10^{-25}\ \text{A}^2/\text{Hz}$, has been evaluated in a series of measurements, performed in the temperature range 1.25–4.2 K, in which the SQUID was very weakly coupled to the resonator.[12] The SQUID output broadband noise as a function of the temperature, as already found by Geng *et al.*[14] for the same SQUID sensor, is well described by a term proportional to the temperature plus a term independent of it. As the term proportional to the temperature is expected,[15] we explain the term independent of the temperature as the preamplifier noise contribution.

On the basis of the S_v measurements presented here, a calculation of the expected back action contribution for the gravitational wave detector AURIGA,[3] which employs the same commercial dc SQUID as the first-stage amplifier, shows that this contribution is negligible as long as the noise matching between the resonant bar and SQUID amplifier is not improved.[16]

Finally, we would like to remark that a direct measurement of S_v permits, without changing the SQUID temperature, a direct evaluation of the energy resolution per hertz also in the case that the SQUID output broadband noise is dominated by the electronics (preamp) contribution; for the SQUID we have tested $\epsilon=S_v/\omega^2 L_i=(1850\pm150)\hbar$ at 4.2 K.

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[10] This is, in general, not true because the sources of both the voltage and current noise are the Josephson junction shunt resistors. This simplification of the model has, however, no practical consequence: one can, in fact, demonstrate that the effect of the noise cross-spectral density is, for the high-quality factors of the resonator, in any case, negligible with respect to the total noise of the resonator–SQUID system.

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